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PLASTIC DEFORMATION OF ROCKS

BY

MUHAMMED ABDUL WAHEB

A

THESIS

submitted to the faculty of the
SCHOOL OF MINES AND METALLURGY OF THE UNIVERSITY OF MISSOURI
in partial fulfillment of the work required for the

Degree of

MASTER OF SCIENCE IN MINING ENGINEERING

Rolla, Missouri

1953

Approved by

W.A. Vinn
Associate Professor of Mining

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INTRODUCTION AND SUMMARY

The purpose of this investigation is to search for a method by which the strength properties of a rock may be described.

For this purpose, an attempt has been made to find out some relationship by which the rocks may be classified or described. For the solution of problems of mine stability, the mechanical properties of a rock should be known, i.e., hardness, toughness and strength. It is difficult to make a study of some method of classification or description of a rock without knowledge of these properties.

With a view to achieve the solution of a general classification for rocks from the strength characteristics, many organizations have obtained information regarding the mechanical properties of rocks. These properties were obtained by different methods and various technicians and seem to give comparable results; but no correlation has been carried out between the fundamental properties to evolve a relationship which will describe a rock. However, it is extremely difficult to find a single property of a rock which will describe every other rock, and it is not easy to determine several such properties which will form a key to describe a rock at different locations. The strength properties of a rock is determined from standardized experimental procedures and specifications. The experimental

results differ much from the calculated theoretical values. This is due to the fact that the strength properties were determined on many assumptions. It is assumed to establish the theory of elasticity that a material is elastic, isotropic, and homogeneous; infinitely divisible without change in properties; and that it conforms to Hook's law, which states that "stress is proportional to strain." Actually none of these assumptions are true. Very careful experiments show that there is some permanent deformation, and Hook's law is not perfectly true for any stress, however small. This shows deviations from the values calculated by a theory and the results obtained by observed data.

The objective of finding a relationship by which a rock can be described has considerable importance, as we are confronted with problems which are difficult to control. Hence a predetermination of conditions are necessary for solution of a particular problem.

Some of the strength properties of a rock can be determined from the stress-strain diagram of the rock. With a view to find a common property which may describe a rock, stress-strain diagrams of different rocks were plotted. It was found that the stress-strain diagrams have different shapes. The diagrams of the same shape are grouped into one family; then, in a similar way, two other families were formed. Then equations of curves of these families were determined by applying the theory of measurements. It was found that each family gives approximately an equation of

one kind. From these equations, and applying the general methods of mechanics, the centroid of the area beneath each diagram has been determined. This point, which is always a fixed point, gave hints as to the nature of deformations and the energy absorbed by the rocks at the point of failure. It will be discussed later in the paper how the positions of the centroids of areas beneath different curves and the equations are used to make a correlation of describing a rock.

REVIEW OF LITERATURE

Strength Properties of Rocks and Methods of Determination

Apparent Specific Gravity

The apparent specific gravity is a measure of the specific gravity of a rock when in a water saturated state; i. e., all interstitial void space in the rock has been filled with water.

Specimens

Diamond drill cores are used. The ends should be squared in the cut-off saw and all surfaces should be smooth and regular. A minimum of three⁽¹⁾ specimens should be pre-

(1) American Society of Testing Materials, 1949.

pared.

Laboratory Equipment

Equipment for these experiments will consist of:

- a. Scale accurate to 0.01 gm.
- b. Immersion tanks and baskets or other means for supporting the specimens in water while being weighed.

Procedure

Specimens will be weighed as air-dried to the nearest 0.01 gm. Then the specimens are saturated and surface-dried and weighed again.

The specimens will then be immersed in distilled water

for 48 hours, after which, while still immersed, they are again weighed.

Calculation

The apparent specific gravity is found as follows:

$$\text{Apparent specific gravity} = A' / B - C^{(2)}$$

-
- (2) Obert, L., Windes, S. L., Duvall, W. J., Standardized Tests Determining Physical Properties of Mine Rock, R. I. #3891, Aug., 1946.
-

where: A' = weight of the air-dried specimen
 B = weight of the saturated and surface-dried specimen in air.
 C = weight of the saturated specimen suspended in water.

Absorption Test

This test is designed to provide a relative measure of the absorption properties of rocks.

Specimens

Specimens for the test should be of a regular form and have smooth surfaces. ⁽³⁾ Sandstone diamond drill cores will

-
- (3) American Society of Testing Materials, 1949.
-

be used; irregular ends can be prepared with the cut-off saw. At least three specimens shall be prepared.

Equipment

Equipment for the experiment will consist of:

- a. Scales accurate to 0.01 gm.
- b. Immersion tank.

Procedure

Specimens will be dried for 24 hours in the oven at a temperature of 105° C. At the end of this period they should be cooled at room temperature for 30 minutes, then weighed to the nearest 0.02 gm.

After weighing, the specimens will be immersed in distilled water for 48 hours. At the end of this period they should be removed from the water bath one at a time, surface dried with a damp cloth, and again weighed to the nearest 0.02 gm.

Calculations

The percentage of absorption,⁽⁴⁾ by weight, is calculated as follows:

(4) Obert, L., Windes, S. L., Duvall, W. J., op. cit., #3891.

lated as follows:

$$\text{Percentage of absorption} = \frac{B-A}{A} \times 100$$

where: A = weight of oven-dried specimen.

B = weight of specimen after immersion.

Compressive Strength

Compressive strength measures, in force per unit area, the maximum stress which a material develops before failure when under a compressive load.⁽⁵⁾

(5) Obert, L., Windes, S. L., Duvall, W. I., Standardized Tests For Determining Physical Properties of Mine Rock, R. I. #3891, p. 20.

Specimens

Much care should be taken in the preparation of specimens for the compressive strength test.

Diamond drill cores will be used, the irregular ends of which can be squared in the cut-off saw. The length to diameter ratio should be unity, and specimens cut from round smooth cores. The ends should be cut as perpendicular to the core-axis as possible. (5)

(6) American Society of Testing Materials, 1949.

The ends should then be lapped and tested for smoothness until a water film on the lapped end shows full contact with a glass plate.

A minimum of three specimens should be prepared.

Equipment

Equipment necessary will consist of:

- a. Calipers for dimensioning the specimens.
- b. Testing machines available in the campus lab.
- c. Bearing boards and cardboard squares. (3" x 3")

Procedure

After polishing of the specimens is completed, they should be stored and allowed to air-dry for two weeks, to reach equilibrium.

After drying, measure the diameter of each specimen with the calipers. Take the average of three readings accurate to 0.02 inch for each specimen.

If possible, use a testing machine having a spherically-

mounted loading head⁽⁷⁾ and center the specimen as closely

(7) American Society of Testing Materials, 1949.

as possible in the center of the machine. Place a bearing block on the machine base, center the specimen on the block, then the top bearing block on the specimen. Use the cardboard squares between the specimen and the bearing blocks.

Load the specimen until failure, at a rate not exceeding 100 lbs/sec. Record the maximum load applied and a description of the specimen after failure.

Calculations

The compressive strength for each specimen is calculated as follows:

$$C = W / A^{(8)}$$

(8) Obert, L., Windes, S. L., Duvall, W. I., Standardized Tests For Determining Physical Properties of Mine Rock, R. I., #3891, p. 20.

where: C = the compressive strength in psi.
 W = the load at failure in pounds.
 A = the cross-sectional area of the specimen in square inches.

Tensile Strength

The purpose of this experiment is to determine the maximum stress which the material will develop under an axially-applied tensile load.

Specimen

Diamond drill cores - both NX and EX⁽⁹⁾ - will be used;

(9) American Society for Testing Materials, 1949.

these can be cut to lengths in the cut-off saw. NX cores should be 8" in length and EX cores 5". The ends should be regular, but need not be polished. For correlation, at least three specimens should be used.

Equipment

Equipment for the test should consist of:

- a. Calipers for dimensioning the specimen.
- b. Tensile test grips (NX and EX).
- c. Testing machine available in the campus lab.

Procedure

All specimens should be air-dried for at least two weeks. After drying, measure the diameter at the center of the specimens with the calipers. Take the average of three readings (60°) accurate to 0.01 inch.

Position the grips on the specimens and secure them as tightly as possible; center the grips in the testing machine.

Load the specimens until failure, at a rate not exceeding 50 lbs/sec. Record the maximum load applied and describe the specimen after failure.

Calculations

The tensile strength for each specimen is calculated as follows:

$$T = W/A \quad (10)$$

(10) Obert, L., Windes, S. L., Duvall, W. I., op. cit., p 27.

where: T = the tensile strength in psi.
 W = the load at failure in pounds.
 A = the cross-sectional area of the specimen in
 square inches.

Modulus of Rupture

The modulus of rupture, besides being a measure of the tensile properties of a material, also is indicative of the strain energy which a material can absorb before failure.

Specimens

EX and NX diamond drill cores will be used; ⁽¹¹⁾ using

(11) American Society of Testing Materials, 1949.

the out-off saw cut the EX cores to 4" lengths and the NX cores to 6" lengths. The ends should be regular, and the specimens should be uniform.

At least three specimens should be prepared for this experiment.

Equipment:

- a. Calipers for dimensioning the specimen.
- b. Knife-edges.
- c. Testing machine available on the campus.

Procedure

Specimens should be air-dried. The diameter should be determined to the nearest 0.01 inch.

The knife-edges are then placed in the testing machine - two on the base and one centered on the upper plate. For NX cores the bottom edges are placed exactly 5" apart, and for EX cores 3" apart. The inverted edge should be centered between them.

Place the specimen upon the bottom edges, allowing an even overlap on each end. Apply the load through the inverted edge, at a uniform rate not exceeding 50 lbs/sec. A slower rate is advised for those not familiar with the testing machine.

Record the load at failure, and note the manner and type of failure.

Calculations

The modulus of rupture is found from the equation:

$$R = 8WL / D^3 \quad (12)$$

(12) Obert, L., Windes, S. L., Duvall, W. I., op. cit., pp. 23-24.

where: R = the modulus of rupture in psi.
 W = the load of failure in pounds.
 L = the distance between the supports.
 D = the diameter of the specimen.

Impact Toughness

Impact toughness can be defined as a measure of the energy-absorbing properties of a material. The important effect is one of toughness, and it is known that a tough material can be subjected to much higher stresses than a

ductile material, but fails at a much lower total deformation. Conversely the ductile material will fail at a lower stress but can sustain a proportionately greater impact load before fracturing. The net effect often produces comparable results for widely different rock types.

Specimens

The manner of preparing specimens for this test is identical with that for the compression test. A minimum of three is required, but more may be necessary in achieving reliable results. (13)

(13) American Society of Testing Materials, 1949.

Equipment

Equipment for this test will consist of:

- a. Calipers for dimensioning the specimens.
- b. The cantilever-type machine (loading machine).

Procedure

Adjust the height of the platforms upon which the specimen rests - to the nearest inch while the beam is in the horizontal position. Bolt and secure the correct tup to the beam, and position the platform so that the tup will strike the center of the specimen.

In the test, the vertical distance that the tup falls must be known, and this can be determined in two ways: (1) with the beam raised, measure the distance directly. This can only be done with sufficient accuracy for falls of a few inches. (2) with the tup resting on the specimen, take a

zero reading on the vertical circle. Take a second reading with the beam raised to the desired position, and the difference is the angular movement. The Kinetic energy is found from:

K.E. = weight of the beam and tup at the C.G. multiplied by the vertical fall in feet.

The testing procedure⁽¹⁴⁾ is one of trial-and-error, and

(14) Obert, L., Windes, S. L., Duvall, W. I., op. cit., p. 41.

the individual must use his own judgment in determining what impact load will induce failure. The following suggestions may help:

- a. Place a specimen on the platform and secure it with packing. Use a fall which will not cause the specimen to fail, and make the first drop. (4 to 5 inches of vertical fall should be sufficient.)
- b. If the specimen fractures, repeat at a lower impact load using a new specimen.

If no failure is observed (use a glass in looking for signs of failure) gradually increase the load until failure occurs, continuing to use the original specimen.

- c. With the correct fall now approximately determined, the rest of the specimens can be tested, and minor adjustments made in the height of fall in order to more accurately determine the impact load for the rock type being tested.
- d. Final results should be based only upon single falls on

each specimen; repeated loading until failure can only give approximate values - lower than the true values due to the stresses induced by the repeated blows.

Calculations

The impact toughness is found to be:

$$\text{Impact toughness} = \frac{\text{K.E. (ft-lbs)}}{\text{Cross-sectional area of the specimen (in.)}^2} \quad (15)$$

(15) Obert, L., Windes, S. L., Duvall, W. I., Standardized Tests For Determining Physical Properties of Mine Rock, R. I. #4727, p. 4.

Sclerescope Hardness

In the theoretical sense, hardness is an undefinable property of materials. It is doubtful if it can be termed an original mechanical property, and is probably dependent upon the combined effects of several related properties such as elasticity, plasticity, and strength. In general terms, hardness is usually defined as the resistance of a material to plastic deformation.

Future experiments will demonstrate how hardness can be used as a factor in studying rock failure.

Specimens

NX cores prepared for the compression tests will be used for this experiment. EX cores can be used, but will not give as accurate results unless polished with extreme care. (16)

(16) American Society of Testing Materials, 1949.

Equipment

The only equipment needed for this experiment is the non-recording-type Solerescope.

Procedure

Take 20 hardness readings on each specimen. The points of impact should be spaced uniformly and arranged in two mutually perpendicular lines of 10 readings each. Move the specimen only when the tup is in the "up" position, and be sure that the specimen is flush with the instrument base.

Calculations

The hardness of the specimen is equal to the median value when the readings are arranged in array. ⁽¹⁷⁾

(17) Obert, L., Windes, S. L., Duvall, W. I., op. cit., R. I. #3891, p. 359. Also, R. I. #4727, p. 5.

Example: 23
 32
 34 - hardness for five readings shown.
 37
 41

Coefficient of Restitution

The degree to which a pair of colliding bodies approach perfect elasticity is expressed by their coefficient of restitution, ⁽¹⁸⁾ which is defined as the negative ratio of

(18) American Society of Testing Materials, 1949.

the relative velocity after collision to the relative velocity before collision, or:

$$e = v_2 - v_2^1 / v_1 - v_1^1$$

where: v_1 and v_2 = velocity of the tup
 v_1^1 and v_2^1 = velocity of the specimen

With the sclerescope, the specimen is at rest both before and after the collision, so v_1^1 and v_2^1 are equal to zero.

Therefore:

$$e = -v_2 / v_1$$

where: v_2 = rebound velocity of the tup
 v_1 = striking velocity of the tup

and:

$$v_1 = (2gh_1)^{1/2}$$

where: h_1 = the height of fall

and:

$$v_2 = (2gh_2)^{1/2}$$

where: h_2 = height of rebound (in the minus direction)

whence:

$$e = -v_2 / v_1 = - (2gh_2)^{1/2} / (2gh_1)^{1/2} = (h_2)^{1/2} / (h_1)^{1/2} \quad (19)$$

(19) Shaha, M. N., Dynamics of Particles, Lecture.

where: e = the coefficient of restitution.

Poisson's Ratio

Poisson's ratio can be defined as the ratio of the lateral unit strain to the longitudinal unit strain, under

the condition of uniform and uniaxial longitudinal stress within the proportional limit. Since it is the ratio of two lengths, it is a pure number. It can be expressed by the relation: ⁽²⁰⁾

(20) Murphy, Advanced Mechanics of Materials, 1946, p. 60.

$$\mu = e_t / e_1$$

where: e_t = unit strain in a direction perpendicular to the direction of the applied load.

e_1 = unit strain along the direction of the applied load.

or:

$$\mu = \frac{E}{2G} - 1 \quad (21)$$

(21) Windes, S. L., Physical Properties of Mine Rocks - U.S.B.M., Report of Investigation #4727, Sept. 1950, p.5.

where: E = Modulus of elasticity

G = Modulus of rigidity.

For most rocks, the value of the Poisson's ratio is between 1/4 to 1/5; ⁽²²⁾ however, the maximum value of the

(22) Vine, W. A., Geomechanics, Lectures, 1953.

Poisson's ratio is 1/2.

Consider a unit cube on whose two opposite faces (Fig. 5b) a pair of forces is applied. Let the two equal but opposite forces be P_2 . Let e be the elongation produced when the two opposites act simultaneously. Then the new length in

the direction of the load is given by,

$$1 - e$$

And the new length in a perpendicular direction of the applied force is given by,

$$1 + \mu e$$

Therefore the volume is given by,

$$V = (1 - e) (1 + \mu e)^2$$

But the original volume = final volume = 1, for a unit cube.

Hence,

$$1 = (1 - e) (1 + \mu e)^2$$

Or

$$\mu = 1/2, \text{ neglecting quantities of higher power.}$$

Plastic Flow

When a body is subjected to a load, and the elastic limit is passed, the body increases in length _____⁽²³⁾,

(23) Taylor, Donald W., Fundamentals of Soil Mechanics, John Wiley & Sons, New York, 1948.

i.e., there is a deformation, but the body will not recover the original shape and size upon removal of the load. The body is said to have a permanent "set." On repeated stressing or constant loading for a very long time, the body gets elastically fatigued, i.e., the body loses the capacity of regaining the original shape and size. The yield stress, or fatigue stress, decreases, and the body goes to permanent deformation. This happens for many materials even within the elastic limit. This event is due to plastic flow.

Hysteresis Strain

When a body is subjected to stress, and loading and unloading cycles are made on it, the body does not regain the original shape and size upon removal of the load. If the stress-strain diagram of such a body is plotted for a few cycles, it will be seen that the diagram gradually changes its shape, i. e., strain during unloading is greater than that during loading.⁽²⁴⁾ This lagging of strain behind

(24) Case, John, The Strength of Materials, Edward Arnold Co., London, 1938.

stress is called hysteresis. To explain it more precisely, the stress-strain diagram does not retrace its path indicating thereby a permanent set. This is due to the changes in the kinetic and potential energy of the body during stressing, and, more particularly, due to the heat energy developed as a result of this process of loading. The total strain energy absorbed in the body during loading cycles can be measured by the total area bounded by the stress-strain diagrams per cycle and is called the energy of hysteresis. The deformation produced is known as the hysteresis strain.

Grindability Index

The grindability index has been determined by a comparative method,⁽²⁵⁾ i. e., the material is ground to a certain

(25) Obert, L., Windes, S. L., Duvall, W. I., op. cit., R. I., #3891, p. 47.

specified sieve size per revolution of a particular ball mill.

Laboratory Equipment

- a. Cylindrical cast steel ball mill 12 in. in diameter and 12 inches long.
- b. 142 chrome-steel balls weighing 23.25 lbs. graded in size from 1/2 in. to 1 1/2 in. dia.
- c. Test specimen: -6 + 8 fraction of the test materials.

Procedure

250 cc. of the specified sieve size material dry ground for a number of revolutions sufficient to produce 250 percent circulating load. Then the material is screened and the quantity of undersize was determined. This process is repeated until the number of revolutions necessary to produce 250 percent circulating load is determined and the amount of undersize product in lb. per revolution is constant. The undersize material produced in pounds per revolution is known as the index of grindability and is determined from the average of three runs.

THE THEORY OF ENERGY

It is found that a fractional extension e_1 in the longitudinal direction is accompanied by fractional contractions in the two perpendicular (lateral) dimensions which are equal to one another and proportional to e_1 . Writing e_1, e_2, e_3 for the lateral extensions, we have:

$$e_2 = e_3 = -\mu e_1 = -\mu P / E \quad (1)$$

(1) Murphy, Advanced Mechanics of Materials, 1946, p. 60.

where μ is a non-dimensional constant of the material, known as Poisson's ratio.

Simple and Compound Stress

In Figure 5, three perpendicular directions are distinguished by numerals 1, 2, and 3. The distortion of the first block (a) due to the single longitudinal stress P_1 is given by:

$$e_1 = P_1 / E$$

$$e_2 = \mu P_1 / E$$

In blocks (b) and (c), the conditions are identical except the direction of the applied stress. Hence,

$$e_1 = -\mu P_2 / E = e_3$$

$$e_2 = \mu P_2 / E$$

$$e_1 = e_2 = -\mu P_3 / E$$

$$e_3 = \mu P_3 / E$$

Therefore, when stresses P_1 , P_2 , P_3 are applied simultaneously in block (d), the strains will be given by: ⁽²⁾

(2) Modified after Timoshenko, S. Applied Elasticity, Westinghouse Tech. Night School Press, East Pittsburgh, Pa, p.46L

$$e_1 = \{ P_1 - \nu(P_2 + P_3) \} / E$$

$$e_2 = \{ P_2 - \nu(P_3 + P_1) \} / E$$

$$e_3 = \{ P_3 - \nu(P_1 + P_2) \} / E$$

The Strain Energy Stored up by the Compound Stress System

If the initial dimensions of the block are a, b, c, so that the area of the faces on which P_1 is applied is $(b \times c)$, and the magnitude of the forces involved by P_1 is $(P_1 \times bc)$, and since the faces are a distance "a" apart, the amount by which they separate as a result of the distortion is $(e_1 \times a)$. This separation is the displacement resulting from the tension $(P_1 \times bc)$, applied to the shaded faces. Considering three pairs of faces, the total strain energy is given by:

$$\begin{aligned} U &= 1/2 (P_1 \times bc) \times (e_1 \times a) + (P_2 \times a \times b) \times (e_2 \times c) \\ &+ (P_3 \times ca) \times (e_3 \times b) = \\ &1/2 a \times b \times c (P_1 e_1 + P_2 e_2 + P_3 e_3) \end{aligned}$$

Dividing by $a \times b \times c$, the unstrained volume of the block, the elastic strain energy per unit volume which is stored up by the stresses P_1 , P_2 , and P_3 , acting in combination, is given by:

$$U = \frac{1/2 \left\{ P_1^2 + P_2^2 + P_3^2 - \mu (P_1 P_2 + P_2 P_3 + P_3 P_1) \right\}}{E} \quad (3)$$

(3) Modified after Timoshenko and Goodier, Theory of Elasticity, 1951, pp. 147-148.

THE THEORIES OF FAILURE

Failure of a material usually means the material has fractured or broken, but this is not necessarily always true. A structural member or a body can be said to have failed when it ceases to perform the job for which it was intended. A member made up of a ductile material may fail through excessive distortion or elastic breakdown. In a brittle material, on the other hand, the breakdown of elastic properties usually results in fracture. Where loads fluctuate and are repeated a great many times, failure of ductile as well as brittle materials generally results in fracture,⁽¹⁾ without

(1) Marin, Joseph, Strength of Materials, Macmillan & Co., New York, 1949, p. 230.

visible distortion; this type of failure is termed "fatigue." At high temperatures the very gradual continuous flow of material resulting from too high a stress may after a long period of time result in distortion sufficient to cause failure of the structure; this type of failure is called "creep" failure.

The above paragraph indicates that consideration should be given to both general types of failure and equations relating thereto. Perhaps, the most often used theory of failure is "The Maximum Stress Theory" which simply states that given a body in various states of stress, failure is due to the single stress which is the largest, without reference

to the stresses at angles thereto. In other words when one stress, shear, tensile, or compression, has reached its ultimate value, the body will fail.

"The Saint Venant Maximum Strain Theory" may be applied to the general theory of failure through deformation. This theory states that there is an ultimate unit deformation which cannot be exceeded without rupture, no matter in what way the stresses are applied. In the same way there is an ultimate deformation⁽²⁾ which cannot be exceeded without fail-

(2) Laurson, P. C., and Cox, W. J., Mechanics of Materials, J. Wiley & Sons, Inc., New York, 1950, p. 276.

ure through inability to do the assigned job.

A third theory as stated by Gust deals with the shear stress in a body. This theory states that in a complex stress system failure occurs when the shear stress reaches the value of the shear stress at yielding.

For other theories of failure which may be referred to references is made to "Strength of Materials" by Marin, and as published by MacMillan Company, Mechanics of Materials by Laurson and Cox.

SOURCES OF OBTAINING THE STRESS-STRAIN DIAGRAMS

The diagrams shown (Fig. 1.) on page 26 have been taken from various sources. (1)(2)(3)(4)

-
- (1) Johnson, J. B., Materials of Construction, rewritten by Withey, M. O., and Aston, James, under the title Johnson's Mechanics of Materials, John Wiley & Sons, Inc., London, 1939, p. 256.
 - (2) Terzaghi, Karl, Soil Mechanics in Engineering Practice, John Wiley & Sons, Inc., London, 1948, pp. 108, 296.
 - (3) Saigal, Radhenath, Master's Thesis, Colorado School of Mines, 1952.
 - (4) Greenwald, H. P., Maize, E. R., and Rice, G. S., Studies of Roof Movement in Coal Mines, U.S.B.M., Report of Investigation #3355, 1937.
-

The curves numbers 1, 2, 3, 4, 5, 7, 8, 9, 13, and 16 have been reproduced from Johnson's Materials of Construction,⁽¹⁾

-
- (1) Johnson, J. B., op. cit.
-

on a bigger scale.

The methods used in obtaining these diagrams by the author are ordinary and simple tests for determining compressive stresses from the observed values of the applied loads. Then the deformations produced were calculated.

Curve numbers 17 and 18, are also reproduced from Soil Mechanics by Terzaghi.⁽²⁾ The methods used in obtaining

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- (2) Terzaghi, Karl, op. cit.
-

these curves by the author are highly accurate. Settlement

of clay and medium sand were measured for corresponding applied loads.

Curves numbered 2, 12, 14, 17 are also reproduced from data obtained from a Master's Thesis by Radhenath Saigal, Colorado School of Mines, 1952.⁽³⁾ The method used in ob-

(3) Saigal, Radhenath, op. cit.

taining these curves is very accurate. Deformations in each case is measured by strain gages and the corresponding stresses are calculated from the applied load.

The diagram number 11 is reproduced from Studies of Roof Movement in Coal Mines by Greenwald, H. P., Maize, E. R., and Hartment, Irving.⁽⁴⁾ The method used in obtaining

(4) Greenwald, H. P., Maize, E. R., and Hartment, Irving, op. cit.

data for this curve by the author was "testing a simply supported beam having a concentrated load at the center."

EQUATIONS OF STRESS-STRAIN DIAGRAMS

A group of data taken experimentally or collected from previous works are plotted on plain ruled paper. This plot may give curves which will satisfy some equation of the form $y = kx^n$ where k and n are constants.⁽¹⁾ If we take logarithms

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- (1) Woodman, L. E., An Application of the Theory of Measurements to Certain Engineering Problems, Bulletin #1, Vol. 15, University of Missouri School of Mines & Metallurgy, Tech. Series, June, 1942.
-

of each side of the equation $y = kx^n$, we have $\log y = \log k + n \log x$, which is the equation to a straight line (i.e., $y = mx + b$) with the y intercept equal to $\log k$. In the instance the y variable $\log y$ and x variable is $\log x$. The slope is equal to n . Therefore, if a curve obtained from plotting the variables from a set of data gives a curve that looks like a parabola, assume that it is one. Plot the logarithms of the values and if a straight line curve results, the assumption is correct and $y = kx^n$ is the required solution. To write the equation for the curve, measure the slope of the straight line curve; this value is n of the general equation. The intercept of the straight line graph with the y axis is k of the general equation. Hence the equation will be $y = (\text{y intercept}) x^{\text{slope}}$. (Figs. 6 and 7)

The preceding equation indicates how an equation may be found for a curve plotted as parabola of the form $y = kx^n$ and this method is useful in determining other equations

also. Should the given data plot a curve on a plain graph paper as well as log log paper there is still a possibility that an equation of the form $\log y = mx + \log B$ may be found by the same method. This equation still satisfies the straight line equation $y = mx + b$ mentioned heretofore. Then plot the logarithms of the y value and the integer value of the x function, on semi-log paper. If the resulting curve is a straight line, the curve satisfies the above equation $\log y = mx + \log B$. The measured slope of the line is m and the intercept on the y axis is $\log B$ and $y = \text{antilog of (slope } \cdot x + \log B)$.

THE THEORY OF CENTROID

The theory of centroid is of prime importance in the present investigation. Before we can discuss in the present chapter the theory of centroid to depict the material in a lucid way some of the terms are explained which will be used often in the discussion of the theory of the centroid. Hence the center of gravity and its determination is given below.

As it is well known, the force of attraction of the earth for a particle is called the weight of the particle, where a number of particles constitute a body. The body thus consists of a system of particles having gravitational forces whose resultant in space is called the weight of the body. The position of this resultant weight, when the body is in any position, can be determined by means of the principle of moments. However, the resultant weight always passes through one point in the body for any position of the body. The point mentioned above is known as the center of gravity.

Method of Determination of Center of Gravity

Though in theory and practice, various methods are available for determining the center of gravity of a body, here the general procedure has been used. (Fig. 3) This figure gives a plate-like body whose thickness is t . The resultant weight of such a plate can be determined by con-

sidering the plate to consist of an infinite number of small elements. The weight of each element which consists of a small fraction of the body can be given by the expression, ⁽¹⁾

(1) Higdon, A., and Stiles, W. B., Engineering Mechanics, Prentice-Hall Inc., New York, 1949. pp. 56-60.

$$dW = r t dA$$

where: r = weight per unit volume of the material
 dA = area of the element, i.e., surface area in xy plane of Figure 3.

The total weight of the plate is given by,

$$W = \int r t dA$$

The coordinates of a point on the line of action of the resultant weight can be determined by the principle of moment. The moment of the weight of the element about the x -axis is,

$$dM_x = y dW = y r t dA,$$

and

$$M_x = \int y r t dA.$$

The y -coordinate of a point on the line of action of the resultant is,

$$\bar{y} = M_x / W = \int y r t dA / \int r t dA \dots (1-1)$$

Similarly, the x -coordinate of a point on the line of action of the resultant is,

$$\bar{x} = \int x r t dA / \int r t dA \dots (1-2)$$

Thus the center of gravity of a body is on the line parallel

to the z-axis, through the point given by the \bar{x} and \bar{y} coordinates, when t and r are constants. \bar{z} can be obtained by symmetry, by giving the plate a rotational motion, when either t or r is variable, so that, either x or y axis is vertical. From the above it follows that $\bar{z} = \int z r dV / \int r dV$,

where: $dV = dA dt$.

Centroid

In Figure 3, the plate under consideration is homogeneous and has a constant thickness. The product $t \times r$ in equations (1-1) & (1-2) is constant, and can be taken outside the integral sign. Upon elimination of the factor $t \times r$ from the above equations (1-1) & (1-2), we get the equations,

$$\bar{y} = \int y dA / \int dA$$

$$\bar{x} = \int x dA / \int dA$$

where:

the denominator dA is the surface area of the plate in the $x y$ plane, and the numerator the first moment of the area with respect to x or y axis.

Thus the point which could be easily located by the coordinates \bar{x} and \bar{y} is termed as the centroid of the area of the top surface of the plate, and must lie on the plane of the area. However it may be mentioned the centroids of other geometrical figures, such as volumes, areas, which do not lie in a plane can be located by the principle of moments.

Centroid of Area

As it would be seen from the discussions, the centroid of an area is the same point as the center of gravity of a sheet of material, where the thickness is infinitely small and the body is homogeneous. Figure 4 represents a sheet in x , y , and z plane which has a total area A , and a thickness t . The moment of differential volume ($dV = t dA$) of the sheet, about plane zOy , is $x_1 t dA_1$, and the sum of the moments of the differential volumes can be given by the following expression, (2)

(2) Faïres, Virgil M., Analytical Mechanics, Macmillan & Co., New York, April, 1943, pp. 178-184.

$$V \bar{x} = t A \bar{x} = x_1 t dA_1 + x_2 t dA_2 + \dots \Sigma x t dA.$$

As it is mentioned, the centroid of the volume coincides with the center of gravity when the material is homogeneous and would lie in the plane xOy , in Figure when t , the thickness, is infinitely small. In such a case, the moment arms of the element of areas, dA_1, dA_2, \dots, dA_n , are distances $x_1, x_2, \dots, y_1, y_3, \dots$ to the y and x axes and the term t may be cancelled from each side of the equation. Hence we have,

$$A \bar{x} = x_1 dA_1 + x_2 dA_2 + \dots \Sigma x dA.$$

Or for an area A in $x y$ plane and in integral form the coordinates of centroid are,

$$\bar{x} = \int x \, dA / A, \quad \text{and} \quad \bar{y} = \int y \, dA / A \quad (3)$$

(3) Laurson, P. C., and Cox, W. J, op. cit., p. 392.

where:

x and y are axes, are any two convenient axes
in the plane of the area.

With the application of the above equation in Figure
the total area A can be determined by integration. In the
present investigation, the above method of determination of
the centroid of an area has been employed.

METHODS OF DETERMINATION OF CENTROID

Application of Equations

There are two methods for the finding of a centroid for any shaped area. For a curve for which the equation is known the centroid may be located by the calculus method of first moment, i.e.,

Sample Calculation: Curve number 2.

Now referring to the curve number 2 in Figure 2, we may apply the equation established for this curve $y =$ (y intercept) $x^{(\text{slope})}$, i.e., $y = 13.34 x^{1.34}$ to find the centroid of the area beneath the curve. Consider an element of area of thickness dx and length y as shown in the figure. Then the area is equal to,

$$A = \int_0^{1.2} y \, dx$$

But

$$y = 13.34 x^{1.34}$$

Therefore,

$$\begin{aligned} A &= \int_0^{1.2} 13.34 x^{1.34} \, dx \\ &= \left\{ \frac{13.34 x^{2.34}}{2.34} \right\}_0^{1.2} \end{aligned}$$

$$Q_x = \int_0^{1.2} \frac{y}{2} y \, dx = \int_0^{1.2} \frac{1}{2} y^2 \, dx$$

$$y^2 = 13.34^2 (x^{1.34})^2 = 13.34^2 x^{2.68}$$

Substituting the above in the equation for Q_x , we have the

following,

$$Q_x = \int_0^{1.2} \frac{1}{2} 13.34^2 x^{2.68} dx = \left\{ \frac{13.34^2}{2} \frac{x^{3.68}}{3.68} \right\}_0^{1.2}$$

Therefore,

$$\begin{aligned} \bar{y} &= Q_x / A = \frac{(13.34)^2 \cdot \frac{1.2^{3.68}}{3.68}}{2.34} \\ &= 5.15 \end{aligned}$$

Similarly, \bar{x} can be found from the same relationship of integration. Thus,

$$\begin{aligned} Q_y &= \int_0^{1.2} x y dx = 13.34 \int_0^{1.2} x^{2.34} dx \\ &= \left\{ \frac{13.34 x^{3.34}}{3.34} \right\}_0^{1.2} \end{aligned}$$

Or,

$$Q_y = \frac{13.34 (1.2)^{3.34}}{3.34}$$

Therefore,

$$\begin{aligned} \bar{x} &= \frac{\frac{13.34 (1.2)^{3.34}}{3.34}}{\frac{13.34 (1.2)^{2.34}}{2.34}} \\ &= 0.829 = 0.83 \end{aligned}$$

Hence the centroid of the area beneath the curve number 2 in Figure 2 is (.83, 5.15). The centroid of this curve as measured by the method of symmetry was (.83, 5.39). The second method is more approximate than this method, as will be seen from the discussion of the accuracy of the established equation to the curve number 2, by assuming a form of the curve and an equation, and then the general equation was determined. This equation does not satisfy all the points on

the curve accurately but is a close approximation.

Component Parts

The second method is that known as component parts and is actually a simplification of the first method. To find the centroid of any figure, no matter what its shape, such as rectangles, triangles, circles, etc., for which the centroid is known, multiply the area of the component parts of the figure by the distance from a selected axis, sum up all of these values (M) of moments and divide by the total area of the component parts (A). This value will be the distance from the selected axis to the centroid of the figure.

Sample Calculation: Curve number 2, Figure 2.

(a) From triangle, Figure 2, O B C,

$$\begin{aligned} M_1 &= + \frac{2}{3} \text{ base} \times \text{area} \\ &= 0.8 \times 10.56 = 8.448 \end{aligned}$$

(b) From triangle a B d,

$$\begin{aligned} M_2 &= (-\frac{2}{3} \text{ base} + x) \text{ area} \\ &= -0.568 \times 0.742 = 0.422 \end{aligned}$$

x being the distance to "a" from y axis.

(c) From parallelogram adfe,

$$\begin{aligned} M_3 &= - x (\text{area}) \\ &= 0.48 \times 0.28 = 0.1342 \end{aligned}$$

x being the distance from y axis to centroid of parallelogram.

(d) From triangle O e f,

$$M_4 = - x (\text{area})$$

$$= 0.293 \times 0.18$$

x being the distance from y axis to centroid of triangle.

Therefore the x- coordinate of the centroid of curve number 2 is given below:

$$\frac{M_1 - M_2 - M_3 - M_4}{A_1 - A_2 - A_3 - A_4} = \bar{x} = 0.832$$

Similarly \bar{y} is found to be equal to 5.39

The method of integrating in this particular type of problem contains some source of error due to the equation being an approximation as found by graphical method. The method of component parts can be as accurate as necessary, depending on the scale of the drawing and on the number of parts into which it is split. For the following work the method of component parts was used and accepted after being checked by the method of equation. The diagrams used in this investigation are largely exaggerated; therefore, the author feels that the component parts method is superior to that of integrating the curve equation between limits.

RESULTS AND DISCUSSION

A collection of physical data of mine rocks derived from various authorities has been given in Table I, and includes the data for compressive and shearing strength, Poisson's ratio, and the density of various rocks. Table I reveals that granites have average higher compressive and shearing strength than limestones and sandstones, whereas, limestones have greater average density, and Poisson's ratio than granites and sandstones, the latter having the lower values. However, the values given in Table I, as can be noted from the reference cited, are comparatively old, and in a way, the methods used to get these values are not up to date.

In addition to the properties given in Table I, Table II includes percent apparent porosity, impact toughness, abrasive hardness, Sclerescope hardness, rigidity modulus, grindability index, and percent moisture content. As it will be noted from the figures of Table II, the abrasive hardness, and the sclerescope hardness are higher for granites than for sandstones, whereas the values for limestones and shale are intermediate. Similarly, the grindability index increases as it passes from granites successively to limestones and sandstones. The impact toughness of these different types of rocks increases from shale to granite, limestones having an intermediate position.

TABLE I

Tests of American Building Stone Made at the Watertown Arsenal*

Name of Stone	Class & Location	Weight per cu. ft.	Compression Tests		Poisson's Ratio in/in	Shearing Strength psi
			Strength psi	E - psi		
Granite	Pigeon Hill, Mass.	161.5	19,670	6,666,700	-	1550
	Milford, Mass.	162.5	23,775	6,663,000	0.172	2554
	Milford Pink, Mass.	161.9	18,988	5,128,000	-	1825
Limestone	Tuckahoe, N.Y.	178.00	16,203	3,563,200	0.222	1490
	Marble Hill, Ga.	168.60	11,505	9,090,900	0.294	1332
	Creole, Ga.	170.00	13,466	6,896,500	0.345	1369
Sandstones	North River Blue Stone	-	22,947	5,268,800	-	-
	Worcester, Mass.	136.60	9,762	2,439,000	0.227	1242
	Kibbe, Mass.	133.40	10,363	1,834,900	0.300	1150

* Taken from Johnson's Material of Construction

TABLE II
Physical Properties of Mine Rock*

Kind of Rock	S.G.	ASP.	A.P. (%)	C.S. lb/in ²	T.S. lb/in ²	Rup. lb/in ²	I.T. in/in ²	A.H.	S.H.	Y.M. lb/in ² x 10 ⁶	R.G. lb/in ² x 10 ⁶	P.R.	G.I. lb/rev x 10 ⁻³	% M.C
Granite	2.66	2.5	0.90	32,000	930	2900	4.90	19.0	95	3.97	2.20	0.04	0.85	.9
				to 35,500	to 1100	to 3680	to 5.00			to 6.41	to 2.45	to 0.31		to .74
Lime- stone	2.37	2.6	11.00	10,000	490	1550	1.90	2.60	27	4.84	2.06	0.18	3.3	0.17
					to 570	to 1930	to 2.10			to 5.39	to 2.30	to 0.29		to 15
Sand- stone	2.17	2.6	16.00	5100	105	400	1.10	1.20	20	1.03	0.46	0.14	9.68	0.19
	to 2.35			to 10,000	to 206	to 800	to 1.80	to 1.50	to 31	to 1.62	to 0.70	to 0.17		to 15.4
Shale	2.4	2.6	6.10	11,600	18,500	150	0.90	2.50	34	3.0	1.82	0.92	-	-
	to 2.87					to 4200	to 1.0	to 4.30		to 6.37				

*Bureau of Mines, Report of Investigations #4727 and 3891, by S. L. Windes and Wilbur I Duvall. U. S. Department of Interior, Sept. 1950 and Aug. 1946, pp. 8-11.

Figure I shows the stress-strain diagrams of the rocks described in the foregoing Tables I and II, i. e., granites, limestones, sandstones, shale, cement, clay and sand. Then the centroids of the areas under different curves have been determined according to methods described previously. Each curve has been numbered and the coordinates of centroids for different curves are given in Table III. From the positions of the centroids of different curves in different areas, the positions of the centroids can be grouped into three major divisions. These zones as shown in Figure I are OBCA, BDEF, and DJKI. The positions of centroids of different types of granites and limestones fall within the zone OBCA, that of sandstones fall within the zone BDEF, whereas the zone DKJI covers the centroids of brick, clay, sand, and cement.

The areas beneath different curves, as shown in Table III, determine the amount of energy absorbed by different rocks at failure. In coordination with the strength properties as discussed earlier in this chapter, the areas under different curves reveal the nature of the rock. It is apparent from observation that curves 2, 3, 4, 7, and 8, which are classed among one group, have the same shape. The rocks of this group of curves have high strength, elasticity, and are less deformable, upon the application of stress. This group of curves has the positions of their centroids close to the origin, and all lie within the area OBCA. The centroids lie closer to the origin as the rocks of this group are least deformable under the application of stress. The

TABLE III
Coordinates and Areas under Stress-Strain Diagrams

Kind of Rocks	Curve Number	\bar{x} in/in	\bar{y} lb/in ²	Areas under curves in-lb/in ³ = Energy absorbed	Position of Centroids
Granite	2--Juni Granite	0.832	5.390	9.398	In the lower part of zone OBCA(Fig.1)
	4--Milford Granite	0.965	3.950	8.065	
	7--Milford Pink Granite	1.060	2.650	5.590	
	8--Troy N. H. Granite	1.310	3.000	7.650	
Lime- stones	1--Tuckahoe limestone	0.510	4.100	3.650	Lower part of zone OBCA (Fig.1)
	3--Marble Hill, Ga., lime- stone	0.610	3.000	2.670	
	5--Creole, Ga., limestone	0.730	2.250	3.740	
Sand- stone	9--North River Blue Stone	1.440	3.800	11.390	In zone BDEF (Fig.1)
	13--Worcester, Mass., s.s.	1.620	1.580	4.100	
	14--Novajo Sandstone	1.640	1.400	4.750	
	15--Dakota sandstone	1.310	1.010	2.980	
	16--Kibbe, Mass., s.s.	1.870	1.150	4.550	
	10--Dark-red brick	2.000	4.000	18.450	
	11--Shale	1.665	3.950	20.800	BDEF (Fig.1)
Cement	12--Cement	2.200	2.600	24.640	In zone DKJI (Fig.1)
Sand	17--Medium sand	2.120	0.490	1.30	
Clay	18--Clay	2.100	0.210	1.005	

rocks exhibiting the above properties are classed as brittle, and highly elastic, in the present investigation.

Curves number 11, 12, 17, and 18 have the same general shape. The rocks of these curves show higher and constant deformation for a period under the application of stress, and are the least elastic. Due to higher deformation, the positions of the centroids are the furthest away from the origin. The positions of the centroids of all these curves lie within the zone DKJI except that of curve 11 whose centroid falls in zone BDEA. From the above discussed properties, these rocks are classified as plastic or "flow" rocks. By considering the strength properties, and the positions of the centroids, the rocks of these curves are least brittle, hard, and tough, but the rock of curve 11 is considerably tougher than others.

The rocks of the curve numbers 9, 13, 14, 15, and 16, have properties intermediate between the above discussed group I and group III. The positions of the centroids of these curves fall within the zone BDEA, which is in between zones OBCA and DKJI. So far as the distance from the origin is concerned, these rocks are more deformable, and less elastic and brittle than the rocks of group I, but are less deformable than the rocks in group III. The vertical extent of the area under the curves of these rocks is higher than those of group I and III. These areas are a measure of energy absorbed by these rocks at failure, as discussed before. These rocks, therefore, may be classed as tough. These rocks, therefore, may be considered to form a bridge between the rocks of group I and group III, as regards their

physical properties, and the positions of the centroids of their curves are concerned. Hence the rocks of group I are brittle, and those of group III are plastic. The rocks of group II fall in between and are classified as tough.

To show the mathematical functions of these curves, a few representative curves have been selected from Figure 1, and reproduced in Figure 2. Table IV gives the equations to the different types of curves which have been classified under three major groups, i.e., parabolic concave, straight lines, and parabolic convex. For group I, for which the nature of the curves is parabolic concave, the equation is $y = Kx^n$ and represents the rocks of brittle type. The calculated value of the equation which satisfies curve number 2 of this group is $y = 13.4 x X^{(1.34)}$. Similarly the plastic or flow rocks give the curves which are parabolic convex the form of equation which is $y = mx + \log B$. The above equation can be written as $\log y = 0.342 x + \log 5.6$, which satisfies curve number 11. In other words, if the equation for the curve of a particular type of rock is known, and the centroid of the area beneath the curve can be located in x and y coordinates, its nature can be predicted and it can be described.

TABLE IV
Equations to the Curves*

Groups of Stress-Strain Diagrams	Shape	Assumed Form of Equations	Approximate Equations Determined E_2	Remarks
1,2,3,4,5,7, 8,9,13,16	Parabolic concave	$y = Kx^n$	$y = 13.4 x^{1.34}$ (1)	Only curve #2 is satisfied by equation E_2 (1)
9,10,13,14, 15,16	Straight line	$y = mx + C$	$y = mx + C$ (2)	All these curves can be plotted as straight lines by approximation to satisfy equation E_2 (2)
11, 12, 17, 18	Parabolic convex	$\log y = mx + \log B$	$\log y = .342 x + \text{LOG } 5.6$ (3)	Only curve #11 satisfies Equation E_2 (3)

*See Figures 6 and 7.

CONCLUSIONS

- 1) If the shape of the curve of a certain type of rock is known, or an equation of such a curve is determined and the centroid of the area beneath the curve be located, then from the equation and the position of the centroid one may know the characteristics of the rock and its behavior without considering other properties.
- 2) Rocks may be classified into three groups from the consideration of their strength properties and the positions of the centroids of the areas under the stress-strain diagrams.
- 3) From such a classification made by the consideration of the strength properties and the positions of the centroids of the area under the stress-strain diagrams, it was found that granites and limestones are brittle; sandstones, bricks and shale are tough; and clay, sands, and concrete are plastic rocks.

APPLICATIONS

The findings of the present investigation are of great significance in applying to many of the mining and building operations as will be discussed here in short.

The mining engineers, and the mining geologists and prospectors, are concerned with the behavior of rocks. The relationship by which a rock could be described, such as the locations of the centroids of the areas beneath the stress-strain diagrams and the established equations for using for this purpose, may be used by the above mentioned industrialists. The rock properties are to be thoroughly known before developing a mine structure and its operations for reasons of reducing costs, safety, and time. This is evident from the consideration of drilling, blasting, and sinking operations of a mine structure. When the properties and nature of a mine rock is known, methods and costs of operation could be advantageously designed and calculated. Similarly the safety factor of such operations could be determined which would be of great value in the design of mine structures. As it is evident cumulatively much time will be saved which is one of the main factors of consideration in all the operations.

SUGGESTIONS FOR FURTHER WORK

The data from which the stress-strain diagrams have been drawn were as observed by different authors by using different methods. From time to time, considerable improvements have been made in the equipments used in testing the mine rocks for obtaining the stress-strain diagrams. So it is apparent that the homogeneity of the experimental conditions used by different authors is questionable. Hence it is suggested that:

- (1) Standard stress-strain diagrams for different mine rocks should be obtained.
- (2) To obtain such standard diagrams, the rocks should be tested with precise equipment under standard conditions.
- (3) To facilitate the analysis of data and to derive the relations, a good number of rocks should be tested from different localities, belonging to different geologic formations.

The generalizations that have been achieved by the present investigation to correlate the different properties for describing a mine rock could be further established and substantiated from the standard values.

APPENDIX

Elasticity:

The term elasticity is very difficult to define as volumes have been written during past centuries on elasticity. It is the foundation of the theory and practice of mechanics of materials. The stress can be determined by mathematical analysis. In so doing it is assumed that material is elastic, isotropic, homogeneous, and infinitely divisible without change in properties. Further it is assumed that all materials conform to Hook's law, which states that "stress is proportional to strain." In practice, however, we find that none of these assumptions are strictly true, and there is always a deviation from Hook's law.

The shape and volume of a body is changed by the application of a force, torque, or pressure. If the body regains its original shape and size after removal of the applied load the body is called perfectly elastic. Such bodies are non-existent in nature. But we assume that every body is perfectly elastic in our study of the strength properties. The determination of such properties is mainly associated with the deformation produced in the body by the applied force in different manner.

Stress:

It is defined as the force per unit area of a stressed body. This force is exerted by either of the two parts of a

solid upon the other across an imaginary plane of separation. This force has different nomenclature as regards its line of action and direction. It is called a (1) shear stress when the force acts parallel to the plane of separation; (2) normal stress when the force acts normal to this plane; (3) compressive stress, when the normal stress has the direction towards the part acted on by the force; and (4) tensile stress which is opposite to the compressive stress.

The stress is expressed by the following relation:

$$dS = dF / dA$$

$$\text{Lt. } \underbrace{dA}_{\rightarrow} = 0$$

where dS = the unit stress, dF = force applied to the infinitesimal area dA .

Unit Strain:

It is defined as the increase in length per unit length when the applied force is tensile; if the force applied is compression, the unit strain will be a shortening of the length per unit length, whereas, an angular strain is called a shear strain.

Plasticity:

It is defined as the ability of a solid body to retain deformation permanently without fracture. Different rocks and materials have different degree of plasticity. This property of a body depends on its structure and texture factor and the type of the deforming load. A good number of solid bodies have appreciable plasticity under steady load for a sufficiently longer period of time.

Ultimate Strength:

It is defined as the maximum stress of any kind that the body can sustain before failure. It is calculated on the basis of the original dimensions of the body and the final load.

Proportional Limit:

This is the maximum stress that a material can withstand to satisfy Hook's law.

Endurance Limit:

This is defined as the maximum stress that may be repeated during different cycles of loading and unloading without producing fracture of the body.

Modulus of Elasticity:

It is the mathematical expression of Hook's law, and may be defined as the ratio of the stress to the strain produced in the body. It is expressed by the relation:

$$E = \text{stress} / \text{strain}$$

Modulus of Rigidity:

It is the ratio of the unit shear stress to the unit shear strain. Usually it is expressed as:

$$G = \tau / \theta$$

where: G = modulus of rigidity

τ = shear stress

θ = angular or shear strain produced by shear stress .

Modulus of Resilience:

It is defined as the energy per unit volume absorbed by

the deformed body with-in the elastic limit. This energy is produced when a system of uniform forces act on the body uniaxially. This energy will be recovered after the withdrawal of the applied load.

Modulus of Rupture:

It is a fictitious stress usually developed on the extreme fiber of a tension or compression member at the moment the member fails. It is expressed by the following relation:

$$S = M c / I$$

where: M = the maximum bending moment, which causes the fracture

I/c = the section modulus of the member.

Strain Energy:

It is defined as the mechanical energy per unit volume of a stressed body, within the elastic limit. As it is recoverable at the withdrawal of the applied load, it is called elastic recovery. This has important bearing on the study of the mechanics of materials. It is measured by the amount of work done per unit volume of the body by the external impressed forces.

Principal stress:

When a body is stressed three perpendicular planes can be visualized to pass through a point within such a body. The stress on each of these planes are normal, compression or tension. This point defines these planes as the principal planes at the point. The stresses which act on these planes are called the principal stresses. The principal

stress may be more precisely defined as the stress on a shearless plane. In such a state, one of the stresses must be maximum, one the minimum, and the remaining one the intermediate stress.

Creep:

It is defined as the continuous increase in deformation under constant or decreasing stress. This term has importance in the study of metals which reveals, to some extent, the behavior of metals at high temperature under load. It has relationship with the condition of a body under compressive stress when it yields. In the later case the body is said to have plastic deformation. The energy of relaxation is associated with the creep of a body, as the rate of creep is usually zero for practical purposes. Creep is limited within the elastic limit and is recoverable. But many materials undergo failure even within the elastic limit due to creep. When the applied load is gradually removed by maintaining constant deformation, the energy released is called the energy of relaxation.

Rock:

A rock is anything that makes up one of the structural units of the earth's crust.

Mine Rock:

All the rocks and materials which are of concern to a mining engineer are called mine rocks.

Brittle:

Brittle rocks have little or no plasticity. Such mate-

rials show little deformation beyond the elastic limit and will therefore fail suddenly.

Tough:

Tough rocks will withstand heavy shocks or will absorb a large amount of energy before failure.

Ductile:

When a material is capable of being drawn in the form of a wire it is called ductile.

Porosity:

The property of a rock of containing interstices without regard to size, shape, interconnection or arrangement of openings. It is expressed as percentage of total volume occupied by interstices.

Coefficient of Viscosity:

The amount of force necessary to maintain a unit difference in velocity between two layers of water a unit distance apart.

Abrasive Hardness:

The abrasive hardness is determined by using cylindrical rock specimen which is abraded against a rotating steel disc covered with an abrasive. Abrasive hardness is measured from the reciprocal of the weight of material lost per unit area per revolution.

BIBLIOGRAPHY

Books

- Case, John, Strength of Materials, Edward Arnold Co., London, 1938.
- Cutshall, Chester S., Mechanics of Materials, John Wiley & Sons, Inc., New York, 1936.
- Faires, Virgil M., Analytical Mechanics, Macmillan and Co., New York, April, 1943.
- Fairman, Sibert, and Cutshall, Chester S., Mechanics of Materials, John Wiley & Chapman, New York, 1953.
- Higdon, A., and Stiles, W. B., Engineering Mechanics, Prentice-Hall Inc., New York, 1949.
- Laurson, P. C., and Cox, W. J., Mechanics of Materials, J. Wiley & Sons, Inc., New York, 1950.
- Love, A. H. E., A Treatise on the Mathematical Theory of Elasticity, Dover Publication, New York, 4th Ed., 1944.
- Marin, Joseph, Strength of Materials, Macmillan & Co., New York, 1949.
- Murphy, Glenn, Advanced Mechanics of Materials, 1946.
- Nadai, A., Plasticity, Macmillan Books Co., New York.
- Seely, Fred B., Advanced Mechanics of Materials, John Wiley & Sons, Inc., New York, August, 1931.
- Terzaghi, Karl, Theoretical Soil Mechanics, Chapman & Hall, Ltd., London, Dec., 1942.
- Terzaghi, Karl, Soil Mechanics in Engineering Practice, John Wiley & Sons, Inc., New York, 1948.
- Timosenko, S., Applied Elasticity, Westinghouse Technical Night School Press, East Pittsburgh, Pa., 1946.
- Timosenko, S., and Goodie, Theory of Elasticity, 1951.

Taylor, D. W., Fundamentals of Soil Mechanics, John Wiley & Sons, New York, 1948.

Withey, M. O., and Aston, James, Johnson's Mechanics of Materials, John Wiley & Sons, Inc. London, 1939.

Bulletins

American Society of Testing Materials, 1949.

Greenwald, H. P., Maize, E. R., Rice, G. S., Studies of Roof Movement in Coal Mines., U. S. Dept. of Int., U. S. Bureau of Mines, Report of Investigation #3355, June, 1952.

Panek, Louis A., Centrifugal Testing Apparatus for Mine Structure Stress Analysis, U. S. Dept. of Int., U. S. Bureau of Mines, Report of Investigation #4883, June, 1952.

Windes, S. L., Obert, L., Duvall, W. I., Standardized Tests for Determining Physical Properties of Mine Rocks, U. S. Dept. of Int., U. S. Bureau of Mines, Report of Investigation #3891, August, 1946.

Windes, S. L., Physical Properties of Mine Rocks, Part II, U. S. Dept. of Int., U. S. Bureau of Mines, Report of Investigation #4727, September, 1950.

Woodman, L. E., An Application of the Theory of Measurements to Certain Engineering Problems, Bulletin #1, University of Missouri, School of Mines & Metallurgy, Vol. 15, Tech. Series, June, 1942.

Saigal, Radhenath, Master's Thesis, Colorado School of Mines, 1952.

Lectures

Livingston, C. W., Barodynamics, Lecture in class of 1952, Colorado School of Mines.

Shaha, M. N., Dynamics of Particles, Lecture in class of 1940, Univ. of Calcutta.

Vine, W. H., Geomechanics, Lecture in class of 1953, Missouri School of Mines & Metallurgy.

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